Accurate Determination of Frequency Dividers Operating Bands

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Abstract—**In analog frequency dividers, phenomena such as harmonic synchronization and jumps between coexisting quasiperiodic and frequency divided paths must be taken into account in order to accurately predict their operation ranges. This has been done in this letter by means of a detailed bifurcation analysis. A simple simulation technique is also proposed for determining the second harmonic synchronization locus. A MMIC frequency divider has been thoroughly analyzed, obtaining an excellent agreement with measurements.**

I. Introduction

THE global stability of frequency dividers by two has been analyzed in the literature [1], [4] by obtaining their bifurcation loci in a two-parameter plane, often input power P_{in} and frequency ω_{in}. Their division ranges are determined by tracing the I-type bifurcation locus on the $[P_{\text{in}}\text{-}\omega_{\text{in}}]$ plane. An I-type bifurcation corresponds to the transformation from a periodic regime of fundamental ω_{in} to a periodic regime of fundamental $\omega_{\text{in}}/2$ [2]. This prediction of the operating ranges will not be valid if the circuit initially operates in a quasiperiodic regime. This is the normal situation for harmonic injection dividers, which behave as free-running oscillators, this giving place, for relatively low input power values to a quasiperiodic regime, with two fundamentals: ω_{in} and the perturbed autonomous frequency ω_a. From this operation mode, the start of the frequency divider regime will be due to the synchronization of the component at the frequency $2\omega_a$ to the external generator frequency ω_{in}. As the input power is further increased, complex jump and hysteresis phenomena may take place, due to the coexistence of quasiperiodic and I-bifurcated frequency divided paths.

In this letter, a bifurcation analysis is used to provide a deep understanding of the different phenomena leading to frequency division. A simple simulation method is also provided for determining the second harmonic synchronization locus. These techniques have been applied to MMIC frequency divider by two, obtaining very good agreement with the experimental results.

II. Frequency Divider Analysis

Let us consider a frequency divider circuit operating in an autonomous quasiperiodic regime. As the input power or input frequency are modified (system parameters), different phenomena may lead to the frequency division:

A. Autonomous Frequency Extinction

The Hopf bifurcation locus provides the set of ω_{in}-P_{in} points for which an autonomous frequency appears or disappears [2]. However, the transformation from a quasiperiodic regime into a periodic one is generally due to a turning point in the quasiperiodic path [4]. This turning point is responsible for the jump to another branch that may correspond either to a multiplier or to a divider regime, depending on the input generator conditions.

B. I-Type Bifurcation

Let us consider now the input generator values for which the autonomous fundamental has been extinguished and the circuit behaves in a multiplier periodic regime of fundamental ω_{in}. In these conditions, an I-type bifurcation will take place in the system if, for some parameter value, two complex-conjugate natural frequencies $\sigma \pm j\omega_{\text{in}}/2$ cross the imaginary axis to the right half of the complex plane [2]. The fundamental frequency is now divided by two.

C. Phase-Locking

Considering again an initial autonomous quasiperiodic regime, a ratio r may be defined between the two fundamentals: $r = \omega_a/\omega_{\text{in}}$ [3]. As the circuit parameters P_{in}, ω_{in} are modified, this number may reach a rational value, this corresponding to a phase-locking taking place in the system [3]. Frequency division by two will be obtained for P_{in}, ω_{in} values associated with $r = 1/2$. If the circuit is analyzed through harmonic balance H_b, the determinant Δ of the jacobian matrix will take a zero value at the synchronization points, due to the system degeneration. The second harmonic synchronization locus will be given by

$$\begin{aligned} H_b(P_{\text{in}}, \omega_{\text{in}}, \omega_a, \bar{X}) &= 0 \\ \Delta(P_{\text{in}}, \omega_{\text{in}}, \omega_a, \bar{X}) &= 0 \\ r = \frac{\omega_a}{\omega_{\text{in}}} &= \frac{1}{2} \end{aligned} \tag{1}$$

where X are the H_b independent variables.

From a periodic analysis, these are turning points of the solution path, also associated to a zero value of the system determinant (creation of the synchronized states [2]). If a measuring probe [1] at $\omega_{\text{in}}/2$ is introduced into the circuit, its value $A_p e^{j\phi_p}$ will also exhibit a turning point for the same

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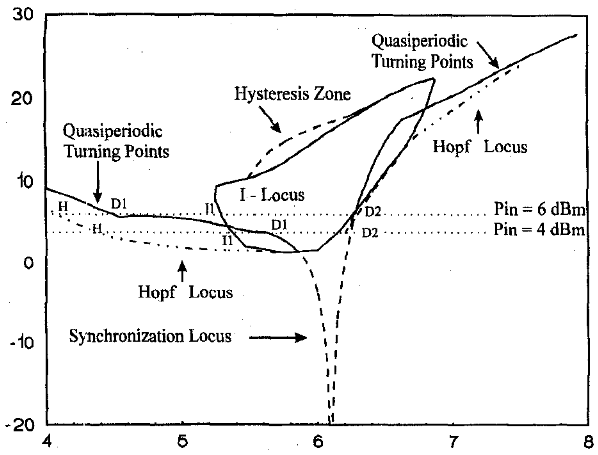

Fig. 1. Bifurcation loci.

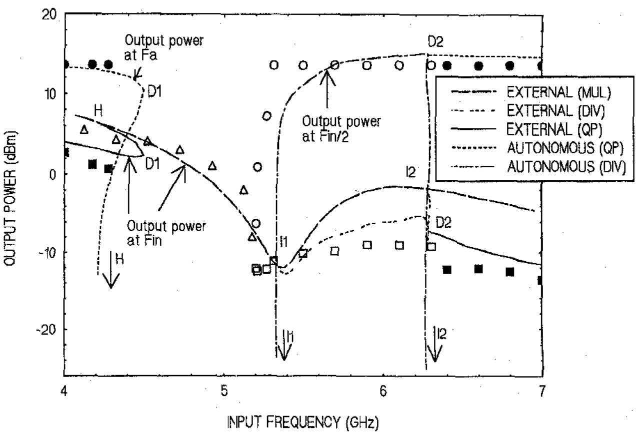

Fig. 2. Bifurcation diagram for $P_{\text{in}} = 6$ dBm. The superimposed points ($\circ, \bullet, \square, \blacksquare, \triangle$) are experimental; $^\circ$ output power in divider regime at $F_{\text{in}}/2$, $\bullet$ output power at F_a in QP regime, $\square$ output power at F_{in} in divider regime, $\blacksquare$ output power at F_{in} in QP regime and $\triangle$output power in multiplier regime at F_{in}.

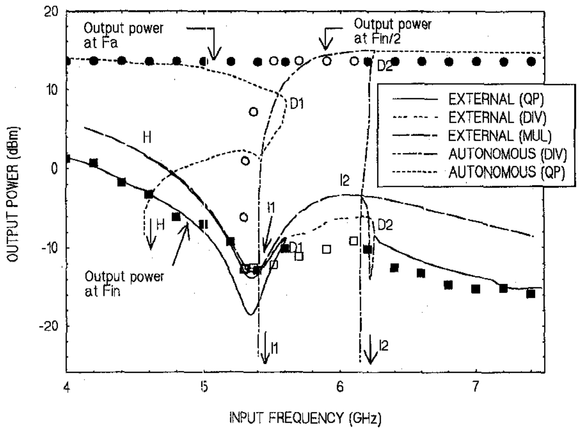

Fig. 3. Bifurcation diagram for $P_{\text{in}} = 4$ dBm. The superimposed points are experimental, filled points correspond to quasiperiodic regime and empty points correspond to frequency division.

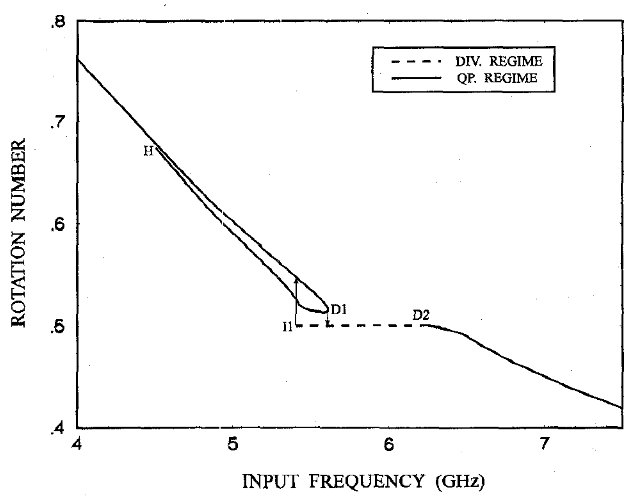

Fig. 4. Rotation number versus input frequency for $P_{\text{in}} = 4$ dBm.

parameters value. Considering the dependence of the H_b state variables on the probe value and the circuit parameters, the locus composed of all the turning points in divided regime may be obtained in a very simple way, by solving the system

$$P(\omega_{\text{in}}, P_{\text{in}}, A_p, \phi_p) = 0$$

$$\det P(\omega_{\text{in}}, P_{\text{in}}, A_p, \phi_p) = \begin{vmatrix} \frac{\partial P^r}{\partial A_p} & \frac{\partial P^r}{\partial \phi_p} \\ \frac{\partial P^i}{\partial A_p} & \frac{\partial P^i}{\partial \phi_p} \end{vmatrix} = 0 \qquad (2)$$

where P is the probe voltage to current ratio (current source) or current to voltage ratio (voltage source) [1].

Below the Hopf bifurcation locus, this curve corresponds to the second harmonic synchronization locus, given by (1). Above the Hopf bifurcation locus, it indicates hysteresis points.

III. Application to a MMIC Divider

The global stability of a MMIC divider by two [1], [4], biased at $V_{\text{GS}} = -0.8$ V, $V_{\text{DS}} = 3$ V has been investigated (Fig. 1). Some of these loci were previously obtained for $V_{\text{GS}} = -1$ V [4]. Now, the second harmonic synchronization locus has also been traced. The intersection between the quasiperiodic turning point curve and the I-locus must also be noticed.

For $P_{\text{in}} = 6$ dBm (Fig. 2) and frequency below 4.5 GHz the circuit behaves in an autonomous quasiperiodic regime. When the turning point $D1$ is reached, a jump takes place to the multiplier branch. If the input frequency is still increased along the multiplier branch, a frequency division, due to an I-type bifurcation, takes place for the input frequency $F_{\text{in}} = 5.327$ GHz (I1). The frequency divider regime goes on along the divider branch until synchronization is lost for $F_{\text{in}} = 6.279$ GHz ($D2$).

For $P_{\text{in}} = 4$ dBm (Fig. 3) and frequency below 5.608 GHz, the system solution is quasiperiodic. From $F_{\text{in}} = 5.405$ GHz there is also a frequency division path. The quasiperiodic path at ω_a and this divided branch intersect, but the solution continues to be quasiperiodic until the turning point $D1$ is reached. At this point, a jump takes place to the divided branch, frequency division starting. If the input frequency is now decreased, the divider regime goes on until the I-type bifurcation is encountered for $F_{\text{in}} = 5.405$ GHz (I1). This hysteresis phenomenon has been confirmed experimentally. The evolution of the ratio r is shown in Fig. 4.

IV. Conclusion

In this letter, some analysis techniques for an accurate prediction of the operating ranges of frequency dividers are provided. These techniques take into account harmonic synchronization and the possible coexistence of quasiperiodic and frequency divided paths, for some parameter ranges. The proposed method has been successfully applied for the complete and accurate characterization of a MMIC divider by two.

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